The History of Pythagoras and his Theorem

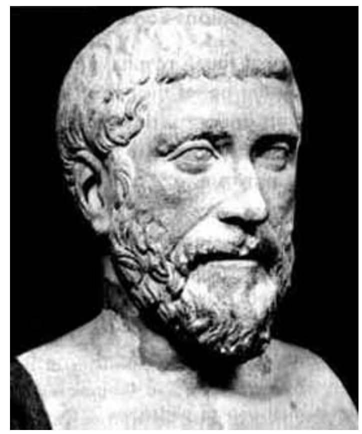
In this section you will learn about the life of Pythagoras and how it is that the theorem is known as the Pythagorean Theorem.

Be aware that there are no good records about the life of Pythagoras, so the exact dates and other issues are not known with certainty. In addition, the names of some of the people as well as the places where Pythagoras lived may have different spellings.

Pythagoras was born in the island of Samos in ancient Greece[١]. There is no certainty regarding the exact year when he was born, but it is believed that it was around ٥٧٠ BC that is about ٢،٥٧٠ years ago! Those were times when a person believed in superstitions and had strong beliefs in gods, spirits, and the mysterious. Religious cults were very popular in those times.

Pythagoras' father's name was Mnesarchus and may have been a Phoenician. His mother's name was Pythais. Mnesarchus made sure that his son would get the best possible education. His first teacher was Pherecydes, and Pythagoras stayed in touch with him until Pherecydes' death. When Pythagoras was about ١٨ years old he went to the island of Lesbos where he worked and learned from Anaximander, an astronomer and philosopher, and Thales of Miletus, a very wise philosopher and mathematician.

Thales had visited Egypt and recommended that Pythagoras go to Egypt. Pythagoras arrived in Egypt around ٥٤٧ BC when he was ٢٣ years old. He stayed in Egypt for ٢١ years learning a variety of things including geometry from Egyptian priests . It was probably in Egypt where he learned the theorem that is now called by his name.



Pythagoras of Samos

By the time he was about ٥٥ years old he returned to his native land and started a school on the island of Samos. However, because of the lack of students he decided to move to Croton in the south of Italy.

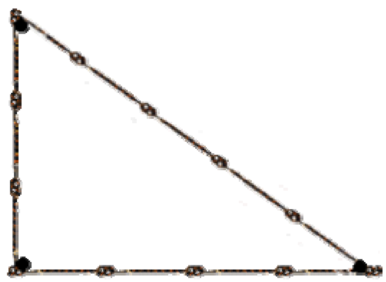
In Croton he started a school which concentrated in the teaching and learning of Mathematics, Music, Philosophy, and Astronomy and their relationship with Religion. It is said that as many as ٦٠٠ of the worthiest people in the city attended the school, including Theana whom he married when he was ٦٠. The school reached its highest splendor around the year ٤٩٠ BC. He taught the young to respect their elders and to develop their mind through learning. He emphasized justice based on equality. Calmness and gentleness were principles encouraged at the school. Pythagoreans became known for their close friendships and devotion to each other. More than anyone before him Pythagoras combined the spiritual teachings with the pursuit of knowledge and science.

Pythagoras also headed a cult known as the secret brotherhood that worshiped numbers and numerical relationships. They attempted to find mathematical explanations for music, the gods, the cosmos, etc. Pythagoras believed that all relations could be reduced to number relations.

At some point Pythagoras was exiled from Croton and had to move to Tarentum. After ١٦ years he had to move again, this time to Metapontus where he lived four years before he died at the age of ٩٩.

Here we have a picture of a statue of Phytagoras in the island of Samos. If you click on the figure you'll be able to see a larger picture. On the bottom of the statue the text is "ΠΥΘΑΓΟΡΑΣ Ο ΣΑΜΙΟΣ". The literal translation is "Pythagoras the Samosan", but the preferred translation is "Pythagoras of Samos".

Now let's talk a bit about the theorem that bears his name. The Egyptians knew that a triangle with sides ٣, ٤, and ٥ make a ٩٠° angle. As a matter of fact, they had a rope with ١٢ evenly spaced knots like this one:



That they used to build perfect corners in their buildings and pyramids. It is believed that they only knew about the ٣, ٤, ٥ triangle and not the general theorem that applies to all right triangles.

The Chinese also knew this theorem. It is attributed to Tschou-Gun who lived in ١١٠٠ BC. He knew the characteristics of the right angle. The theorem was also known to the Caldeans and the Babylonians more than a thousand years before Pythagoras. A clay tablet of Babylonian origin was found with the following inscription: "٤ is the length and ٥ the diagonal. What is the breadth?"

So why is it called the Pythagorean Theorem? Even though the theorem was known long before his time, Pythagoras certainly generalized it and made it popular. It was Pythagoras who is attributed with its first geometrical demonstration. That is why it is known as the Pythagorean Theorem. There are hundreds of purely geometric demonstrations as well as an unlimited (that is right -- an infinite number) of algebraic proofs.

The Pythagorean Theorem is one of the most important theorems in the whole realm of geometry. We will conclude this section by stating the theorem in words:

> *The square described upon the hypotenuse of a right-angled triangle is equal to the sum of the squares described upon the other two sides.*

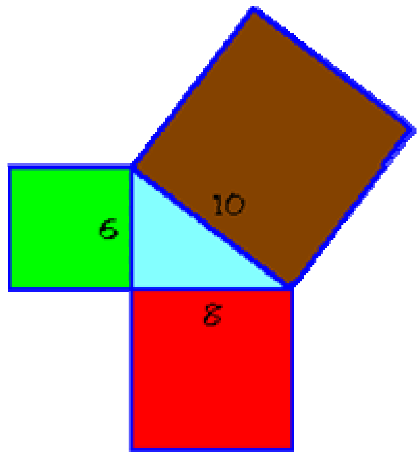
Another way of saying the same thing is:

> *When the two shorter sides in a right triangle are squared and then added, the sum equals the square of the longest side or hypotenuse.*

Pythagorean Theorem Problems

١) The Pythagoras Proof:

This is a hands-on exercise for you to convince yourself that the Pythagorean Theorem works. It is based on the actual proof that is attributed to Pythagoras. On the following figure we have a right triangle with a square associated with each of its sides:



Using the dimensions associated with the three sides; calculate the area of each of the squares. Then make sure that the area of the hypotenuse's square (brown) equals the areas of the other two squares together.

Now for the hands on part; draw an equivalent picture on a piece of paper. You can use any size triangle as long as it is a right triangle. Cut up and reassemble the two small squares to form a square identical to the larger one.

٢) The classical ladder problem:

There is a building with a ١٢ ft high window. You want to use a ladder to go up to the window, and you decide to keep the ladder ٥ ft away from the building to have a good slant. How long should the ladder be?

٣) Baseball diamond:

On a baseball diamond the bases are ٩٠ ft apart. What is the distance from home plate to second base in a straight line?

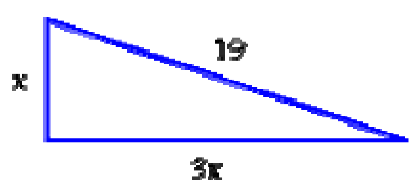
٤) Equilateral triangle:

An equilateral triangle has vertices at (٠،٠) and (٦،٠) in a coordinate plane. What are the coordinates of the third vertex? You may want to sketch it out.

Note: The sides of an equilateral triangle are identical in length.

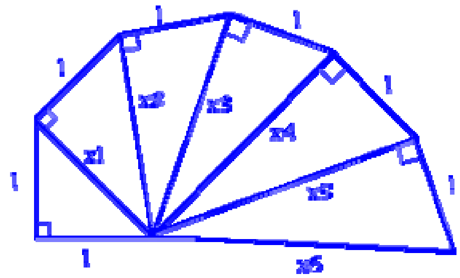
٥) An algebraic problem:

Find out the length of sides a and b on the following triangle:



٦) An iterative problem:

Look at the following figure. Start by finding the value for $X_{١}$, then for $X_{٢}$, then $X_{٣}$, and so on until you get the value for $X_{٦}$. Write the lengths as square roots, as that makes it simpler.



What is the value of $X_{٦}$?

٧) A ٣D problem:

We have a wooden box that measures ٤ ft. by ٣ ft. by ٢ ft.:

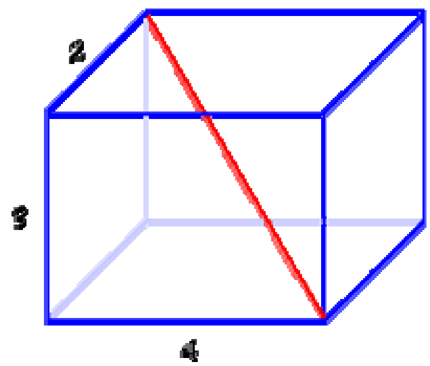


Figure out: What is the longest straight pole, like the red one, that you can have inside the box?

٨) Pythagorean Triples:

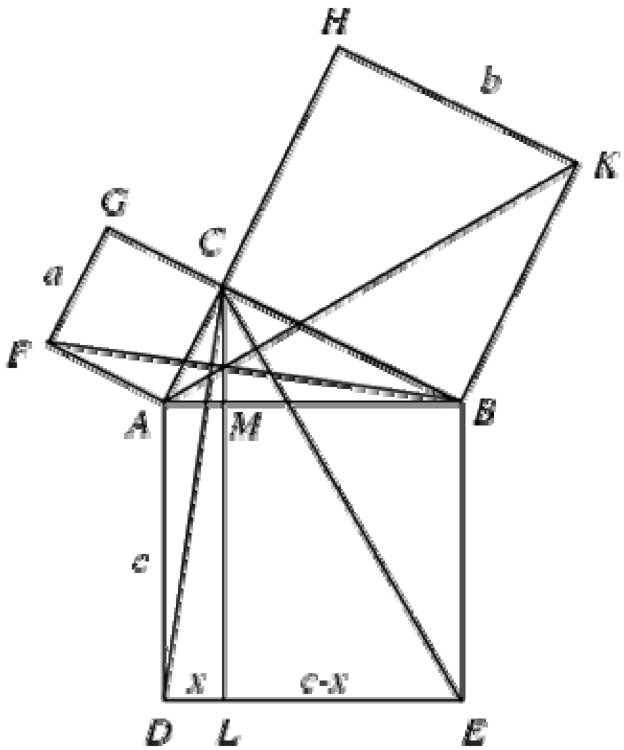
The Pythagorean Triples were described with Tip number ١. Here we will describe a method to generate all of the Pythagorean Triples. There is a simple formula that gives all the Pythagorean triples. If m and n are two positive integers and m < n, then the triples can be generated with the following equations:

$$a = n^{٢} - m^{٢}$$
$$b = ٢mn$$
$$c = n^{٢} + m^{٢}$$

It's easy to check algebraically that the sum of the squares of a and b is the same as the square of c.

Proof of Pythagorean Theorem

Perhaps the most famous proof of all times is Euclid's geometric proof (Tropfke ١٩٢١ab; Tietze ١٩٦٥, p. ١٩), although it is neither the simplest nor the most obvious. Euclid's proof used the figure below, which is sometimes known variously as the bride's chair, peacock's tail, or windmill. The philosopher Schopenhauer has described this proof as a "brilliant piece of perversity" (Schopenhauer ١٩٧٥; Gardner ١٩٨٤, p. ١٥٣).



Let $\triangle ABC$ be a right triangle, $\square CAFG$, $\square CBKH$, and $\square ABED$ be squares, and $CL \parallel BE$. The triangles $\triangle FAB$ and $\triangle CAD$ are equivalent except for rotation, so

$$2\triangle FAB = 2\triangle CAD.$$

Shearing these triangles gives two more equivalent triangles

$$2\triangle CAD = ADLM.$$

Therefore,

$$\square ACGF = ADLM.$$

Similarly,

$$\square BC = 2\triangle ABK = 2\triangle BCE = BL$$

so

$$a^2 + b^2 = cx + c(c - x) = c^2.$$

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